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A SEMI-CONTINUOUS BOX COUNTING METHOD FOR FRACTAL DIMENSION MEASUREMENT OF SHORT SINGLE DIMENSION TEMPORAL SIGNALS-Preliminary study

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Box counting method allows to measure the eventual fractal dimension (D) of a single dimension temporal signal. However its accuracy varies as a function of the frequency sampling (Fs) and the duration of the tested signal (Sd). Consequently, as it is impossible to highly increase Fs, this method is not suitable for short physical signals D measurement. Thus, we designed a semi-continuous box counting method (SCBC) allowing a better approach of the small scales of the signal, especially useful in case of short single dimension temporal signal.

Let N = number of samples of the tested signal. SCBC provides with the first M points of the graph log – log owing to the dyadic division of boxes at large scales up to a certain box size S_M, such as $S_M = 2^M/Fs$. Then, at smaller scales, for each successive point the box size decreases by 1/Fs, that provides the – log with a large number of points. Thus, when $N/S_{(M+x)}/Fs$ is not a whole number, the analyzed signal is peripherally and symmetrically reduced in abscissa and ordinate, so that a whole number of boxes is obtained. But these truncated samples are then reintroduced for designing following boxes. Using SCBC we measured D of mathematical signals which D is known, and compared these results to those obtained using the classic dyadic box counting method.

1 Introduction

A continuous physical signal such as a sound signal, of which speech is an example, constitutes a time series. If this series is self-similar, measurement of its fractal dimension (D) allows fluctuations to be characterised by quantifying the complexity and irregularity of this signal. The classic box counting method seems the most appropriate to quantify complexity in the temporal succession of events. As the box size is divided by 2 for each measurement, this method may be termed dyadic box counting method (DyBC). In the framework of our research to improve signal processing in auditory prosthesis [1,2,3], we believed that this eventual D of speech elements could help patients with implants to recognize speech without having to lip-read.

In a recent study [10] we carried out a fractal approach of vowels. But, unlike mathematical signals, a physical signal cannot be fractal at every scale. However, let be N(r) the number of boxes filled by events at the resolution r, by definition the generalised dimension D_{gen} of the graph of the signal amplitude vs time is obtained using the limit

$$D_{gen} = \lim_{r \to 0} [\log(N(r))] / [\log(1/r)]$$

Thus, in our study, we had to use the higher resolution to approach infinitely small time scales, and to estimate the eventual fractality of vowels. In this aim we could observe that it is possible to appreciate, at least, the tendency of the points set by calculating the slope of the last 3 points, i. e. D of small size boxes (ssD). We demonstrated that:

- in case of sinusoidal signal for which $D_{gen} = 1$, ssD allows a better approach of the value of D_{gen} than using all points for the slope calculation;
- in the particular case of speech, ssD is able to supply a kind of significant signature of the vowels signal.

However we noted that the accuracy of DyBC varies as the ratio of [frequency sampling / frequency of the tested signal]. But we observed also that this accuracy depends on the duration of the tested signal. Indeed several authors, e. g. Robinson [5], estimate that at least 10 points are necessary to characterise the eventual fractality of a physical object. However, let N be the sample number of the tested signal, p be the number of points on the log -log graph, obeying $N=2^p$, then one may observe that:

- p drastically diminishes when N decreases
- 1024 samples - i. e. for example 64 ms if Fs = 16 kHz - supply only 10 points; therefore these data characterise the shortest signal which may be studied by DyBC with Fs = 16 KHz.

Consequently, as it is impossible to highly increase Fs, DyCB is not suitable for the measurement of the dimension D of short signals. Thus, we designed a semi-continuous box counting method (SCBC) allowing to approach the small scales in case of short single dimension temporal signal. In this study, we describe this SCBC. Then, testing several mathematical signals, either with a known dimension, or which are not fractal, we compare its results in various cases to those obtained using the DyBC.

2 Methods and Material

2.1 Measurement methods

Let Ts be the sample duration i.e. 1/Fs. In this study, in order to first compare DyBC and SCBC, we used the lowest value allowing use of DyBC

- for Brownian signals: Fs=44.1 kHz, Sd=23 ms,N=1024
- for sinusoidal signal: Fs=16 kHz, Sd=64 ms, N=1024 i.e

Then, in order to appreciate the SCBC efficiency, we use various short duration signals (0.36, 0.72 and 1.45 ms for Brownian signals, and 1, 2 and 4 ms for sinusoidal signal) and compare the measured D with the theoretical D.

2.1.1 The dyadic box counting method

This method used 10 boxes whose sizes vary from 23 ms (1024 Ts) to 0.045 ms (2 Ts) for Brownian signals, and 64 ms (1024 Ts) to 0.125 ms (2 Ts) for sinusoidal signal.

We measured the slope on the log-log graph of these 10 points, which include all scales from 1024 Ts to 2 Ts (asD); we studied also the small scales from 8 Ts to 2 Ts, measuring only the slope of the 3 last points (ssD) on the log-log graph.

2.1.2 The semi-continuous box counting method

This method is directly derived from DyBC. It provides the first M points of the graph owing to the dyadic division of boxes at large scales up to a certain box size S_M, such as

$S_M = 2^M/Fs$. Then, at smaller scales, for each successive point the box size decreases by 1/Fs, that provides the log-log graph with a large number of points. Thus, when ($N/S_{(M+x)}/Fs$) is not a whole number, we have to peripherally and symmetrically reduce the analysed signal in the abscissa and the ordinate, so that a whole number is obtained. But these truncated samples are then reintroduced for designing the following boxes.

In the first part of this study, M=6, allowing to obtain a whole set of 36 points (Table I). We also separately measured asD, which is supplied by the slope of 36 points on the log-log graph , and ssD supplied by the slope of only the last 7 points, corresponding to the small scales from 8 Ts to 2 Ts.

Table I. Sizes of the 36 successive boxes of SCBC in case of D measurement of a 64 ms signal, owing to a 16 kHz frequency sampling, with M=6. For each size (i.e. duration) box, Bs-sn = box size in sample number.

In this example a dyadic division of box size is performed to obtain the first 6 points on the log-log graph, i.e. large scales, from 1024/Fs (64 ms) to 32/Fs (2 ms); then, for small scales, a 1/Fs (0.0625 ms) decreasing is realised from 31/Fs (1.9375 ms) to 2/Fs (0.1250 ms). However, when the ratio number of samples / duration box is not a whole number, the analyzed signal is peripherally and symmetrically reduced in abscissa and ordinate, so that a whole number is obtained. But these truncated samples are then reintroduced in the measurement of the following boxes. From the 36 boxes of this example, this table only indicates some data, and mainly: the values of the last 10 boxes; the values of the box corresponding to the biggest signal reduction of the signal (24 samples); in this case, for instance, for the measurement of this box (1.525 ms), the 12 first and 12 last samples of the 1024 samples which constitute the tested signal have not been analysed.

Dyadic division decreasing		1/Fs decreasing		Part of the signal which is not analysed with the box size Bs-sn
Box duration	Bs-sn	Box duration	Bs-sn	
64 ms	1028			
32 ms	512			
16 ms	256			
8 ms	128			
4 ms	64			
2 ms	32			
		1.9375 ms	31	1 sample
		1.8750 ms	30	4 samples
		1.8125 ms	29	9 samples
		1.5625 ms	25	24 samples
		0.6250 ms	10	4 samples
		0.5625 ms	9	7 samples
		0.5000 ms	8	0 sample
		0.4375 ms	7	2 samples
		0.3750 ms	6	4 samples
		0.3125 ms	5	4 samples
		0.2500 ms	4	0 sample
		0.1875 ms	3	1 sample
		0.1250 ms	2	0 sample
		0.0625 ms	1	0 sample

In the second part of our study, using various short duration signals, M=0 allowing to obtain 15, 31 and 63 points for 0.36, 0.72 and 1.45 ms in case of Brownian signal, and 1, 2, and 4 ms for sinusoidal signal. In these cases the slope of the log-log graph is based on the 10 last points.

2.2 *Tested signals*

We used 4 mathematical signals. Some of them are fractal with a known dimension: there are 3 Brownian signals (D = 1.5, 1.4, and 1.3). Another signal is not fractal (1 kHz sinusoidal signal), but its generalised dimension D_{gen} is known and equal to 1. For each signal we randomly selected 24 trajectories, each of 64 ms duration. We used Matlab software [6] to transform and analyse these sounds into 16 bit "wav" format.

Using both methods we measured each of these 24 D values and calculated the corresponding D mean value and confidence intervals.

2.3 *Statistical study*

Results were studied using the SPSS statistical package. Repeated measures of analysis of variance, using statistical contrasts to perform pairwise comparisons, were used to compare D measures.

Both DyBC and SCBC use the measure of the slope of the line obtained by linear regression on the log-log graph to determine D. However, only one of the two components is a variable, which represents the fluctuation in the signal. The other is a regular function of time. Therefore, in order to quantify the eventual bias, we calculated the slope and the corresponding error, i.e. the difference between the observed ordinate and the theoretical ordinate on the regression line.

3 Results

Measurement of slope and corresponding error gave values ranging from 10^{-4} to 10^{-7}. For each linear regression we observed a very small error ranged from 10^{-4} and 10^{-7}.

Table II. Mean value (upper line) and confidence interval (lower line) of D measurement of 24 randomly selected 64 ms duration parts of mathematic signals, using DyBC and SCBC, as a function of the size of the studied scales (all sizes = asD, from 23 to 0.045 ms for Brownian signals and 64 to 0.1250 ms for sinusoidal signal; only small sizes = ssD, from 0.18 to 0.045 ms for Brownian signal, and 0.5 to 0.125 ms for sinusoidal signal).

	DyBC - asD 10 boxes	SCBC - asD 36 boxes	DyBC - ssD 3 boxes	SCBC - ssD 7 boxes
Brownnian signal D=1.5	1.47 1.45 - 1.48	1.42 1.40 - 1.43	1.34 1.33 - 1.34	1.30 1.30 - 1.31
Brownnian signal D=1.4	1.39 1.37 - 1.41	1.36 ** 1.35 - 1.38	1.27 1.26 - 1.27	1.25 1.25 - 1.26
Brownnian signal D=1.3	1.30 1.29 - 1.33	1.28 1.27 - 1.30	1.21 1.20 - 1.21	1.25 1.25 - 1.26
1 kHz Sinus. signal D_{gen}=1	1.83 1.83 - 1.84	1.85 1.85 - 1.85	1.16 1.09 - 1.23	1.15 1.11 - 1.18

3.1 Fractal signals

Results are summarised in Table II.

In case of asD measurement, for Brownian signal, for which D = 1.5, the difference between DyBC and SCBC is significant. However none of these methods includes the theoretical D value in its confidence interval

For Brownian signals for which D = 1.4 and 1.3, the difference between DyBC and SCBC is not significant.

In case of ssD measurement, the difference between DyBC and SCBC is not significant for Brownian signal for which D = 1.5. It is significant for Brownian signal with D = 1.4 and 1.3. But the theoretical D value is never included in both methods' confidence intervals as the intervals do not overlap.

3.2 Non fractal signal

In cased of asD measurement, the difference between DyBC and SCBC is not significant. The calculated D is very different from D_{gen}.

In case of ssD measurement, the difference between DyBC and SCBC is not significant. But the calculated D tends to approach the D_{gen}.

Table III. Mean value (upper line) and confidence interval (lower line) of D measurement of 24 randomly selected parts of mathematical signals of various duration, using SCBC. NAS = total number of samples which have not been analysed.

	0.36 ms **16 Ts** **15 points** NAS=43	**0.72 ms** **32 Ts** **31 points** NAS=229	**1.45 ms** **64 Ts** **63 points** NAS=920
Brownnian signal **D=1.5**	1.31 1.27 - 1.34	1.33 1.31 - 1.35	1.30 1.28 - 1.32
Brownnian signal **D=1.4**	1.25 1.23 - 1.27	1.30 1.28 - 1.32	1.30 1.28 - 1.30
Brownnian signal **D=1.3**	1.24 1.23 - 1.26	1.28 1.27 - 1.29	**1.29** **1.28 - 1.31**

	1 ms **16 Ts** **15 points** NAS=43	**2 ms** **32 Ts** **31 points** NAS=229	**4 ms** **64 Ts** **63 points** NAS=920
1 kHz Sinus. signal $\mathbf{D_{gen}=1}$	1.32 1.31 - 1.33	1.19 1.18 - 1.20	1.15 1.14 - 1.15

3.3 Short duration signals

The measured D value progressively approaches the theoretical D as the duration of the signal increases (Table III). In case of Brownian signals with D = 1.4 and 1.3, the differences observed between each pair of measures are significant only for the comparisons 0.36 ms vs 0.72 ms and 0.36 ms vs 1.45 ms. In case of non fractal signal the difference is significant for the comparisons 1 ms vs 2 ms, 1 ms vs 4 ms, 2 ms vs 4 ms.

4 Discussion

4.1 Methodology

We could have tried to study other fractal signals than Brownian signals. However, in practice it is difficult to obtain such mathematical signals, which are surely fractal; this difficulty is evident if, for instance, we consider white noise.

The value of M is important and we shall discuss it later.

4.2 Efficiency of SCBC

4.2.1 Fractal signals

In case of asD measurement, one may observe that the efficiency of both DyBC and SCBC decreases as D - i.e. the signal roughness - increases. That implies that the signal irregularities are not correctly taken into account by the relatively narrow window which constitutes the tested signal duration.

Besides, in case of ssD measurement, despite the difference between SCBC and DyBC being significant, none of these methods gives correct D value. That is probably due to the fact that, in this particular case of fractal signal, the observation of only the small scales does not include the long dependence of the Brownian signal.

4.2.2 Non fractal signal

The values of asD measurement are very different from the true value of D_{gen}. That seems normal, because the concept of D_{gen} is valuable only on infinitely small scales.

Conversely, ssD tends to reach the true value of D_{gen}, and we may observe that there is no significant difference between DyBC and SCBC

4.2.3 Short duration signal

If we consider Table III and the SCBC-asD column of Table II, and recall the significance of the difference between the various pairs of measurements, we may observe that the efficiency of SCBC depends on the signal duration, whatever the signal. In case of Brownian signal, this efficiency also inversely depends on the D value. These discrepancies may be explained by:

- the truncation of the Brownian signal, which distorts its correlation function adding high frequencies in its spectrum,
- the temporary suppression of some samples during the SCBC management. This suppression represents a bias; this bias probably increases with the number of samples which are not analysed, and also with the reduction in signal duration. Here the value of M must be discussed. In this preliminary study we arbitrarily chose M=6 for comparison between SCBC and DyBC, and M=0 for testing efficiency of SCBC in case of signals shorter than 64 ms. However, the fewer are the samples which are not analysed, the better is probably the SCBC efficiency. It could be worthwhile to improve SCBC efficiency choosing M value as great as possible, in such a way as to obtaining only 10 necessary points for slope calculation.

Nevertheless one must underline that, in case of non fractal signal, the value of D_{gen} is correctly approached by SCBC.

4.2.4 ssD significance

One may observe that the use of asD is efficient for signals with long correlations (e.g. Brownian signals), while ssD is more efficient for signals where the dimension is observable at the smallest scales (e.g sinusoidal signals) or for signals that are uncertain to have a fractal dimension. But fractal dimension is always defined as a limit, which implies that the structure can be analysed in arbitrarily high resolution. However, this trend is only achieved for an infinite number of points, which is conceivable for a theoretical signal, but impossible for a natural signal. In practice, if we consider a physical object, its structure is given by a finite data set representing its discrete digitisation. Thus, using only the last points of the curve, i.e the smallest scales, the dimension of an object which is fractal is not measured exactly. But ssD may be considered as a kind of signature of the signal, which may be characteristic [4] in some cases; the advantage of this estimation of ssD is that it highlights the convergence to generalised dimension of an object for the smallest resolutions. That explains why, in case of sinusoidal signal, we observed that the measurement of ssD gives the trend towards the correct D value more rapidly (Fig.1,2,3,4).

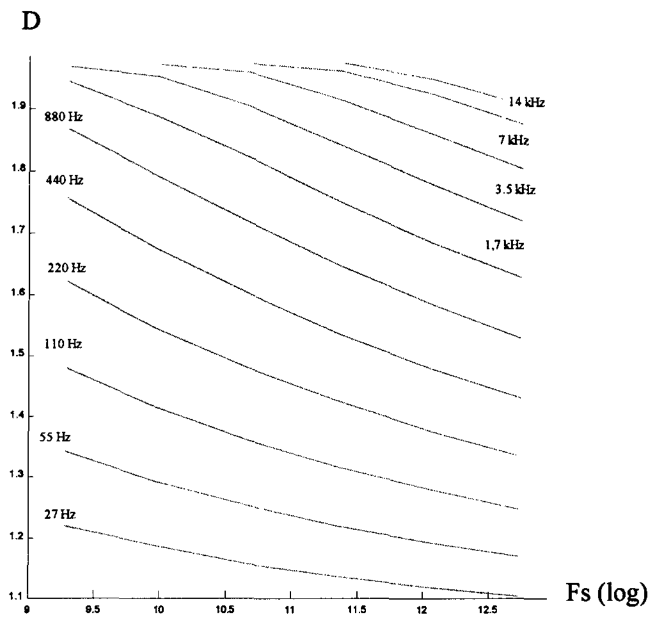

Figure 1. D values (ordinate) obtained using box-counting method for several sinusoidal sounds as a function of frequency sampling using asD measurement.

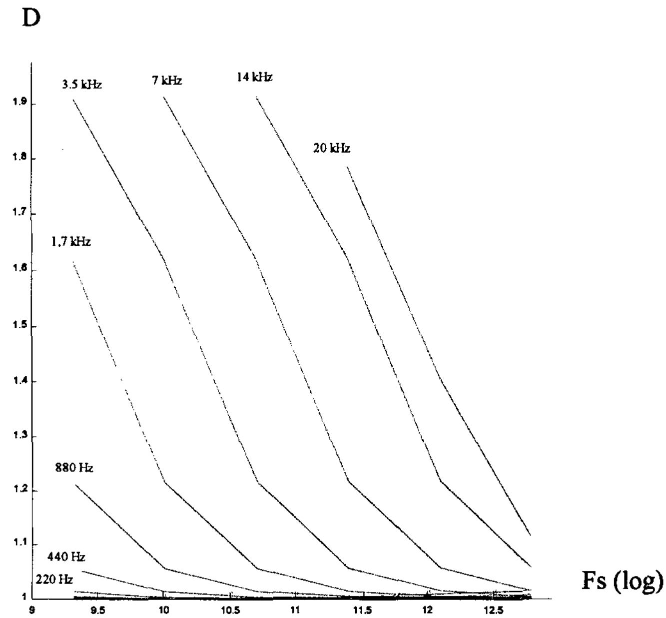

Figure 2. D values (ordinate) obtained using box-counting method for several sinusoidal sounds as a function of frequency sampling using ssD measurement.

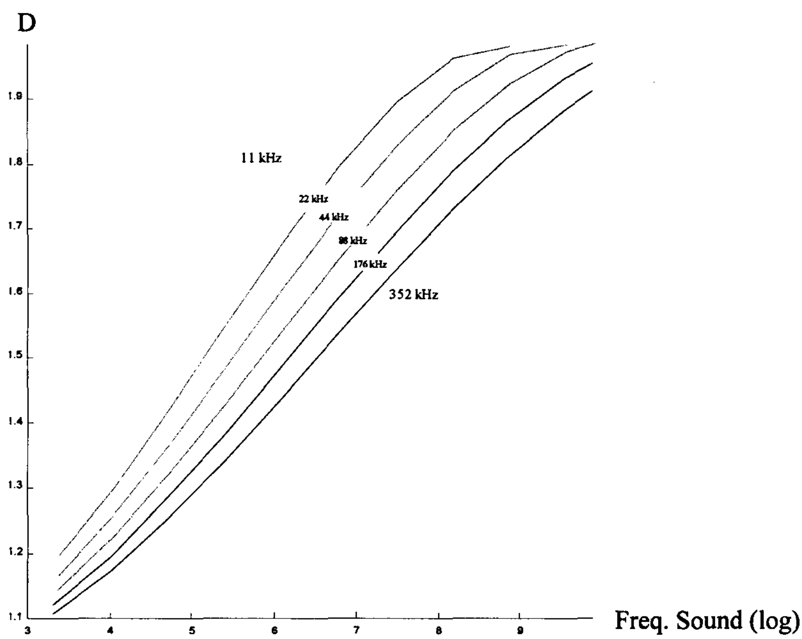

Figure 3. D values (ordinate) obtained using box-counting method for several sinusoidal sounds as a function of frequency sound using asD measurement.

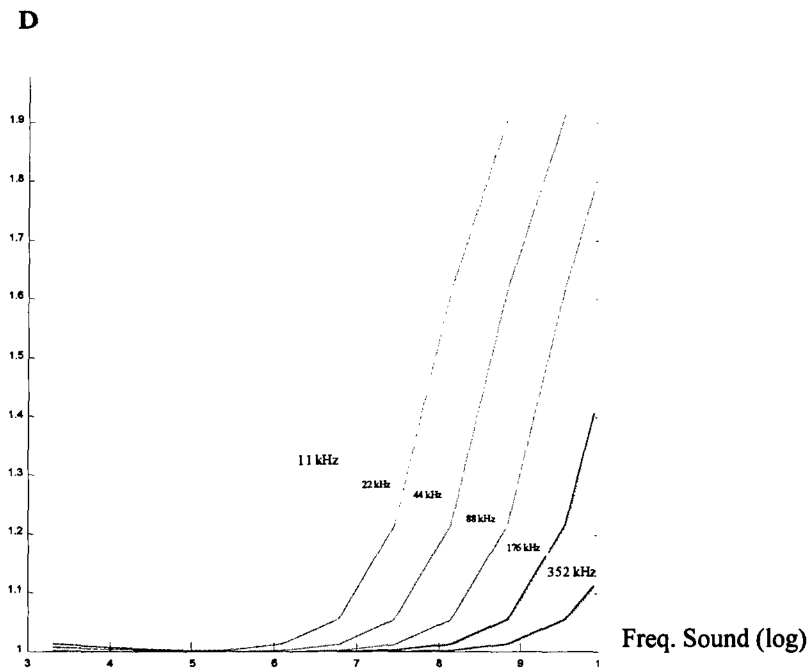

Figure 4. D values (ordinate) obtained using box-counting method for several sinusoidal sounds as a function of frequency sound using ssD measurement.

4.3 In the literature

One may find other methods in the literature to determine D of a single dimension temporal signal. In our Laboratory, we attempted to employ these methods [7] using the same mathematical signals as we used in this paper. All these results may be compared.

4.3.1 The Richardson length method [8]

This method measures the perimeter of an object with various length of rulers; when plotting log perimeter against log ruler length, a fractal object gives a straight line with a negative slope S, and D = 1 - S. But this method is difficult to be computed without any risk of bias. Moreover most signals must be digitised to be easily studied using computers; consequently they are not yet continuous, burepresent a series of data. Anyway Pickover [9] demonstrated that this method supplies with the same results as using box-counting method on the same but digitised signal.

4.3.2 The power spectrum density (PSD) measurement

This technique has been employed [10] using the Fourier transform generated spectral density function, which gives intensity or power at each frequency. The results demonstrate that this method is valuable for sounds with PSD in 1/f, as Brownian signals for instance. But it is not usable for signals analysis, which have not PSD in 1/f.

4.3.3 The rescaled range analysis (RRA)

This is the earliest empirical method. It has been described by Hurst to study the long time series of Nile floods. It is based on the assumption that a large number of natural phenomena are time series with a long-term correlation. This method was recently studied by Bassingthwaighte *et al.* [11]. It allows to characterise a one-dimensional time series by simultaneously providing a measure of variance and long-term correlation of its components (the term « memory » is often used). But the results obtained with the signals where the PSD is not a power law, or which do not have - even theoretically - a fractal dimension are totally incoherent. This is the case for a constant or a sinusoidal signal. RRA only appeared to be effective for the signals with a very long correlation, or a power spectrum of 1/f. However, even in these cases, application of known Brownian signals underestimated true H for H>0.72, and underestimated H for H<0.72.

4.3.4 The dispersional analysis method (DAM)

This method is also empirical. It was recently studied by Bassingthwaighte *et al.* [12] to evaluate whether it was effective to determine the fractal dimension of a Brownian signal, according to the type of this signal and the size of the data set. DAM measures the standard deviation of the single dimension signal at different scales. The objective is to reveal a possible power law in the successive values of the standard deviation of the signal. But for the signals where the PSD is not a power law, or which do not have a fractal dimension, the results are similar to those obtained with RRA. Consequently, despite the fact that the evaluation of true H of Brownian signals is more precise than with RRA, one may conclude that all these methods are less efficient than box DyBC or SCBC.

4.3.5 Other tools

Although the aim of this paper is not the study of the eventual fractal features of speech, one may briefly mention other tools proposed by several authors for use of fractal geometry as a model for describing irregularities of graphical wave forms of human speech. Bohez *et al.* [13] used box counting method and cluster analysis; however Ds values have not been clearly specified. Using different box shapes and a specific box-counting algorithm, Maragos [14] described different D (1 to 1.3) for vowels as a function of their scale analysis, and his results are in accordance with our first measurement [4] . Other authors studied the multifractal and chaotic features of speech, but we shall not discuss them in this paper, which only considers a fractal approach to single dimension, short temporal signals.

5 Conclusion

As SCBC increases the number of points at small scales, it improves the DyBC performances, pushing far away its physical limits due to frequency sampling and duration of the tested signal. However its efficiency presents comparable limits:

- some of them are physical, and for example Fs=16kHz, at least 10 points on the log-log graph are necessary, the shortest signal which may be analysed using SCBC must have a minimum of 0.6875 ms duration.
- others are due to the principle of the method, which leads to some samples not being analysed. However, choosing M value as a function of Sd reduces the number

of points necessary for the slope calculation to 10 on the log-log graph could improve SCBC efficiency.

We plan to study further this possible improvement. Then we shall use this method to explore the eventual dimension D of transient parts of consonants.

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